Incoherence without Exploitability

Brian Hedden

Noûs 47 (3)

1 Introduction

If you believe that your sister is in Tashkent and that Tashkent is in Uzbekistan, but you also believe that your sister is not in Uzbekistan, then your beliefs are not merely odd, but irrational. Moreover, it seems your beliefs are irrational in virtue of their having a certain structure, in particular their being logically inconsistent. It is a requirement of rationality that your beliefs be logically consistent.

But belief is not an all or nothing affair. It comes in degrees. We believe some things more strongly than others. Your levels of confidence, or degrees of belief, are known as *credences*. Are there any rational constraints on the structure of your credences, just as there are rational constraints on the structure of your binary beliefs? Suppose you are certain that your sister is in Tashkent, and you regard it as rather likely that Tashkent is in Uzbekistan, but you also regard it as unlikely that your sister is in Uzbekistan. Your credences are in some sense incoherent and resemble inconsistent beliefs.

Plausibly, you are irrational if you have these incoherent credences, just as you would be irrational in virtue of having inconsistent beliefs. But can we give any *argument* that this sort of incoherence is irrational?

The Dutch Book Argument (DBA) is the most prominent argument that purports to show that it is indeed a requirement of rationality that your credences have a certain structure.[1] It is a requirement of rationality that

[1] Actually, there are a variety of Dutch Book arguments. My focus here is on the Dutch Book Argument for conformity to the probability calculus. My comments will not bear on, for example, the diachronic Dutch Book arguments for conditionalization (Lewis (1999), reported earlier by Teller (1973)) and reflection (van Fraassen (1984)).

your credences be *coherent*, in a sense to be made precise in the next section. For if your credences are incoherent, then you will be predictably exploitable by a clever bookie who knows no more about the world than you do. A Dutch Book is a set of bets that together guarantee you a loss. The idea, then, is that if your credences are incoherent, they will license you to accept each member of some Dutch Book. In this way, a bookie could exploit you by getting you to accept each bet in a Dutch Book, thereby guaranteeing himself a gain and you a loss.

There are many objections one might raise to the DBA. Is it irrational to be predictably exploitable? And even if it is irrational to be predictably exploitable, is this irrationality distinctively *epistemic*, rather than merely pragmatic? What if you have an aversion to gambling or don't care about money, so that you wouldn't want to engage in any betting behavior whatsoever?

Defenders of the DBA have presented thoughtful responses to all of these objections.[2] But there is a deeper problem with the DBA. I will argue that, even if you have no aversion to gambling and care only about money, it is possible to have incoherent credences without being predictably exploitable, provided that your credences are incoherent in a very particular way. Therefore, even if we accept that predictable exploitability is irrational, the DBA can at best show that *some* incoherent credences are irrational. It cannot be used to condemn all incoherent credences as irrational. As such, it is only a partial result. I close by arguing that if it is possible to be incoherent at all, then some structural constraints on credences must be brute requirements of rationality that we accept on the basis of their intuitive plausibility rather than on the basis of any compelling argument.

2 The Canonical Dutch Book Argument

To make the DBA precise, we first have to say more about what we mean by saying that certain credences are coherent or incoherent. Assuming that

[2] See in particular Christensen (1996), who argues that the thrust of the DBA is not that if you have incoherent credences, you might lose money, but that if you have incoherent credences, you are inconsistent in how you evaluate the fairness of bets, and this is a distinctively epistemic sort of inconsistency. So, it is immaterial whether there are actually any clever bookies around, or whether you actually dislike gambling. Skyrms (1987) suggests a similar interpretation of the argument. My comments on the DBA will apply equally to their non-pragmatic interpretation.

your credences can be represented by numbers, they are said to be coherent just in case they conform to the probability calculus. That is, given a set S of propositions (taken to be sets of possible worlds) closed under complementation and union, your credences are coherent if and only if they obey the following three axioms[3]:

> **Non-Negativity**: $P(A) \geq 0$ for all $A \in S$
>
> **Normalization**: $P(T) = 1$ for all necessary truths $T \in S$
>
> **Finite Additivity**: $P(A \vee B) = P(A) + P(B)$ for all disjoint $A, B \in S$

Next, we have to be more precise about the notion of predictable exploitability and what it is for your credences to license you to perform certain actions. To say that your credences license an action is just to say that, supposing those credences were rational, it would be rationally permissible for you to perform that action. You are predictably exploitable just in case your credences license you to accept each member of some Dutch Book.

Then, we can put the DBA in the following form:

> **P1**: If your credences are incoherent, then they license you to accept each member of a Dutch Book.
>
> **P2**: If your credences license you to accept each member of a Dutch Book, then it is irrational to have those credences.
>
> **C**: It is irrational to have incoherent credences.

Why believe Premise 1? Here, the DBA focuses on the notion of a *fair betting quotient*. Your fair betting quotient for A is defined as the number n such that your credences license you to accept either side of a bet on A at $n : 1 - n$ odds (i.e. to accept a bet which pays \$$(1 - n)$ if A and \$$(-n)$ if $\neg A$ and also a bet which pays \$$(n - 1)$ if A and \$$n$ if $\neg A$).[4]

Then, the DBA supports Premise 1 with (i) an assumption that your fair betting quotients match your credences (so that for all A and n, having

[3]This is using the Kolmogorov (1933) axiomatization.

[4]Sometimes, the notion of a fair betting quotient is explicated in more behavioristic terms, as the number n such that you are in fact disposed to accept either side of a bet on the relevant proposition at $n : 1 - n$ odds. As behaviorism has fallen out of favor, I explicate the notion of a fair betting quotient in normative terms, as involving the odds at which your credences license you to take either side of the bet.

credence n in A licenses you to accept a bet which pays \$$(1-n)$ if A and \$$(-n)$ if $\neg A$ and also a bet which pays \$$(n-1)$ if A and \$$n$ if $\neg A$), and (ii) the Dutch Book Theorem, proven by de Finetti (1937), which states that if your credences give rise to fair betting quotients that are incoherent (i.e. are negative, non-finitely-additive, or not 1 for each necessary truth), then they license you to accept all the bets in some Dutch Book. The Dutch Book Theorem goes as follows: [5]

Dutch Book Theorem for Non-Negativity

Suppose that your fair betting quotient for A is n, where $n < 0$. Then, by definition, you are licensed to accept a bet which pays you \$$(n-1)$ if A and \$$(n)$ otherwise. But this bet guarantees a loss of at least \$$(-n)$.

Dutch Book Theorem for Normalization

Suppose that your fair betting quotient for some necessary truth T is n, where $n < 1$. Then, by definition, you are licensed to accept a bet which pays \$$(n-1)$ if T and \$$n$ otherwise. But since T is a necessary truth, this bet guarantees you \$$(n-1)$, which is negative.

Now, suppose your fair betting quotient for T is m, for some $m > 1$. Then, you are licensed to accept a bet which pays \$$(1-m)$ if T and \$$(-m)$ otherwise. But again, this guarantees you \$$(1-m)$, which is negative.

Dutch Book Theorem for Finite Additivity

Suppose that your fair betting quotient for A is x, your fair betting quotient for B is y, and your fair betting quotient for $A \vee B$ is z, where A and B are disjoint. If $z < x + y$, then you are licensed to accept a bet which pays \$$(1-x)$ if A and \$$(-x)$ otherwise, a bet which pays \$$(1-y)$ if B and \$$(-y)$ otherwise, and a bet which pays \$$(z-1)$ if $A \vee B$ and \$$z$ otherwise. These bets together yield \$$(-x-y+z)$ no matter what, but since $z < x+y$, this is negative and hence a loss for you.

Suppose instead that $z > x + y$. Then you are licensed to accept a bet which pays \$$(x-1)$ if A and \$$x$ otherwise, a bet which pays

[5] Here my explication closely follows that of Hajek (2008).

\$$(y-1)$ if A and \$$y$ otherwise, and a bet which pays \$$(1-z)$ if $A \vee B$ and \$$(-z)$ otherwise. These bets together yield \$$(x+y-z)$ no matter what, but since $z > x+y$, this is negative and hence a loss for you.

The Dutch Book Theorem and the assumption that your fair betting quotients match your credences together entail Premise 1.

Premise 2 is pretheoretically compelling. There seems to be something irrational about credences which license you to accept a set of bets that logically guarantees you a loss. But there would be a worry about Premise 2 if *all* credences, whether coherent or incoherent, licensed you to accept Dutch Books. However, the Converse Dutch Book Theorem (proven by Kemeny (1955) and Lehman (1955)) states that if your credences are coherent, then they do not license you to accept any Dutch Books.

To summarize, Premise 1 is supported by the assumption that fair betting quotients match credences and the Dutch Book Theorem. Premise 2 is supported by intuition and the Converse Dutch Book Theorem (which forestalls a potential objection to Premise 2). Premises 1 and 2 together entail the conclusion that it is not consistent with your being rational that you have incoherent credences.

3 How Credences License Actions

The DBA relies on specific claims about the bets that your credences license you to accept. How should we evaluate these claims? We have a natural, compelling general framework for thinking about which actions, including those related to betting, your credences license you to perform. This framework is known as expected utility theory. Given a set of options available to you, expected utility theory says that your credences license you to choose the option with the highest expected utility, defined as:

$EU(A) = \sum_i P(O_i|A) \times U(O_i)$[6]

[6] P is your credence function, U is your utility function, representing your preferences, and the O_i are outcomes that form a partition of logical space. This is the formula for Evidential Decision Theory. I ignore Causal Decision Theory here since it is more complex, and the distinction between Evidentialism and Causalism is immaterial for present purposes.

On this view, we should evaluate which bets your credences license you to accept by looking at the expected utilities of those bets. Ignoring worries about aversions to gambling and indifference to money (so that your utility function is linear with respect to money), your credences license you to accept a bet just in case that bet has non-negative expected value, given your credences. This is because the alternative option, rejecting the bet, has an expected value of 0, so the option of accepting the bet has an expected value at least as great as that of its alternative just in case the bet's expected value is non-negative. Then, to say that your credences license you to accept each bet in a Dutch Book is to say that each bet in the Dutch Book has non-negative expected value, given your credences. And the claim that incoherent credences license you to accept Dutch Books is the claim that, for each incoherent credence function, there is a Dutch Book, each member of which has non-negative expected value, given that credence function.

As a quick example of how this works, consider Normalization. Suppose you violate Normalization by having credence 0.8 in a necessary proposition T and credence 0.2 in its negation. Then, consider a bet which pays $\$-0.20$ if T and \$0.80 if $\neg T$. The expected value of this bet is $(0.8 \times \$-0.20) + (0.2 \times \$0.80) = \$0$. Since this bet has non-negative expected value, you are licensed to accept it. But since T is a necessary truth, the bet guarantees you a loss of \$0.20.

Now, one might object to this use of expected utility considerations by claiming that expected utility theory only applies to agents with probabilistically coherent credences. Talk of *expected* utility presupposes that we are dealing with a coherent probability function, since the mathematical expectation of a random variable (such as utility) by definition requires the use of a coherent probability function. So, if your credences are incoherent, it doesn't even make sense of talk about the expected utility of an action, relative to your credences.

If this were right, then the DBA would be in serious trouble, since it is unclear how credences could license accepting certain bets in a manner independent of expected utility theory. But I think the objection is misguided. It is not as though the expected utility formula itself requires that we plug in only coherent credences in order to get an output. When we plug in negative credences, or credences greater than one, and the like, we still get a number; we don't end up dividing by zero or anything like that. So even if it is improper to use the term 'expected utility' in the context of an incoherent credence function, we can still talk about the *schmexpected utility* of an action A, which is just the result of plugging in credences to the formula $\sum_i P(O_i|A) \times U(O_i)$, whether or not they are coherent. This is just expected utility, minus the presupposition that we're dealing with a coherent credence function. And importantly, the motivations for employing expected utility theory in the context of coherent credences carry over to employing schmexpected utility theory in the context of incoherent credences. In general, we should judge actions by taking the sum of the values of each possible outcome of that action, weighted by one's credence that the action will result in that outcome. This is a very intuitive proposal for how to evaluate actions that applies even in the context of incoherent credences.

From now on, therefore, I will be assuming that expected utility theory is the last word on how credences license you to accept bets.

4 Negation Coherence and the Limits of the DBA

We saw earlier that a crucial assumption of the DBA is that your fair betting quotients match your credences; i.e. that having credence n in A licenses you to accept a bet which pays $\$(1-n)$ if A and $\$(-n)$ if $\neg A$ and a bet which pays $\$(n-1)$ if A and $\$n$ if $\neg A$ (the other side of the former bet). But given expected utility theory, this assumption fails. We should think of your fair betting quotient for A as the number n such that a bet which pays $\$(1-n)$ if A and $\$(-n)$ if $\neg A$ and a bet which pays $\$(n-1)$ if A and $\$n$ if $\neg A$ each has an expected value of 0 (so that you are licensed to accept either). The expected value of the former bet is:

$$P(A)(1-n) + P(\neg A)(-n)$$

And the expected value of the latter bet is:

$$P(A)(n-1) + P(\neg A)(n)$$

Each of these expected values equals 0 when $n = P(A)/(P(A) + P(\neg A))$. So, your fair betting quotient for A is $P(A)/(P(A) + P(\neg A))$:

Fair Betting Quotients: $FBQ(A) = P(A)/(P(A) + P(\neg A))$

This means that your fair betting quotients match your credences just in case your credences in a proposition and its negation sum to 1 (that is,

$FBQ(A) = P(A)$ just in case $P(A) + P(\neg A) = 1$). Call this constraint 'Negation Coherence':

Negation Coherence: For all A, $P(A) + P(\neg A) = 1$

Therefore, the assumption that your fair betting quotients match your credences presupposes that you satisfy Negation Coherence. If you violate Negation Coherence, you might have credence n in some proposition without being licensed to accept each side of a bet on that proposition at $n : 1 - n$ odds.

Consider the example from the previous section, where you violate Normalization by having credence 0.8 in a necessary truth T. If you satisfy Negation Coherence and therefore also have credence 0.2 in $\neg T$, then a bet which pays $\$-0.20$ if T and \$0.80 if $\neg T$ has an expected value of \$0, and so your credences license you to accept this bet, even though it guarantees you a loss. But if you violate Negation Coherence and assign, say, credence 0.1 to $\neg T$, then this bet has an expected value of $(0.8\times\$-0.20)+(0.1\times\$0.80) = \$-0.08$, and hence your credences do not license you to accept this bet.

The DBA cannot presuppose Negation Coherence without circularity, since Negation Coherence is a consequence of probabilistic coherence. In particular, it follows from Normalization and Finite Additivity.[7] The DBA can still be used to argue that if you satisfy Negation Coherence, then any incoherence elsewhere in your credence function is irrational (i.e. that it is a requirement of rationality that if you satisfy Negation Coherence, then you fully satisfy probabilistic coherence), but it cannot itself show that Negation Coherence is a requirement of rationality.

5 Incoherence without Exploitability

Not only does the assumption that fair betting quotients match credences fail, but there are also cases where your credences are incoherent and yet there is no set of bets, each of which has a non-negative expected value, which together guarantee you a loss. Hence, you are provably invulnerable to Dutch Books. Taking propositions to be sets of possible worlds, consider the following credences in a model with three worlds, w_1, w_2, and w_3.

[7] By Normalization, $P(A \vee \neg A) = 1$. Since A and $\neg A$ are disjoint, Finite Additivity requires that $P(A) + P(\neg A) = P(A \vee \neg A)$.

$P(\{w_1, w_2, w_3\})$	$= 0.9$	(Your credence in the necessary proposition)
$P(\emptyset)$	$= 0$	(Your credence in the necessary falsehood)
$P(\{w_1\})$	$= 0.7$	(Your credence that you are in w_1)
$P(\{w_2, w_3\})$	$= 0$	(Your credence that you are not in w_1)
$P(\{w_2\})$	$= 0$	(Your credence that you are in w_2)
$P(\{w_1, w_3\})$	$= 0.8$	(Your credence that you are not in w_2)
$P(\{w_3\})$	$= 0$	(Your credence that you are in w_3)
$P(\{w_1, w_2\})$	$= 0.8$	(Your credence that you are not in w_3)

The first thing to note is that these incoherent credences yield coherent fair betting quotients. To see this, note that whenever $P(A) \neq 0$ and $P(\neg A) = 0$, your fair betting quotient for A is 1 and your fair betting quotient for $\neg A$ is 0. This is because, as we saw in the previous section, your fair betting quotient for A is $P(A)/(P(A)+P(\neg A))$, while your fair betting quotient for $\neg A$ is $P(\neg A)/(P(\neg A) + P(A))$. Hence, your fair betting quotients in the case above are:

$FBQ(\{w_1, w_2, w_3\})$	$= 1$
$FBQ(\emptyset)$	$= 0$
$FBQ(\{w_1\})$	$= 1$
$FBQ(\{w_2, w_3\})$	$= 0$
$FBQ(\{w_2\})$	$= 0$
$FBQ(\{w_1, w_3\})$	$= 1$
$FBQ(\{w_3\})$	$= 0$
$FBQ(\{w_1, w_2\})$	$= 1$

These fair betting quotients are non-negative, finitely additive, and 1 for the necessary proposition. As a result, the Dutch Book Theorem does not kick in to entail that you are licensed to accept all the bets in a Dutch Book in this case.

We can go further and prove that these credences do not license you to accept any Dutch Books.[8] One way to prove that you are not licensed to

[8] Unfortunately, the Converse Dutch Book Theorem does not itself entail that you are not licensed to accept any Dutch Books in this case. This is because the theorem does not state that if your fair betting quotients are coherent, then you are not licensed to accept a Dutch Book. Rather, it states that if your fair betting quotients are coherent *and* match your credences, then you are not licensed to accept a Dutch Book. In this case, where your fair betting quotients are coherent but don't match your credences, neither the Dutch Book Theorem nor its converse applies. Moreover, it is possible to have incoherent credences that yield coherent fair betting quotients but still lead to exploitability. For instance, in

accept any Dutch Books is to show that there is a world such that you are not licensed to accept any bet which has you losing money if that world is the actual world; that is, that there is a world such that no bet which costs you money in that world has an expected value greater than or equal to 0. In this model, w_1 is such a world. This is because any proposition P which is true in w_1 is such that you assign positive credence to it and zero credence to its negation. Hence, you are not licensed to accept any bets on P which cost you money if P is true, since in the expected value calculation, losses incurred if P is true could not be compensated for by gains made if $\neg P$ is true.[9] So, you are not licensed to accept any bet which costs you money if w_1 is the actual world. Hence, you are not licensed to accept a set of bets which costs you money *no matter which world is the actual world*. Hence, you are not licensed to accept a Dutch Book.

As a final point, it was important that in the case above, you did not have any negative credences; you only violated Finite Additivity and Normalization. This is because we can prove in general that if you have negative credences, then you are licensed to accept a Dutch Book. The reason is that negative credences essentially flip losses to gains in the expected value calculations. Suppose that $P(A) < 0$, leaving open whether $P(\neg A)$ is less than, greater than, or equal to 0. Consider a bet which costs you \$$r$ if A and costs you \$$s$ if $\neg A$. By making r sufficiently large (so that $P(A) \times (-r)$ is large and positive) and making s sufficiently small (so that the absolute value of $P(\neg A) \times (-s)$ is less than $P(A) \times (-r)$), we can make this bet have positive expected value for you, despite the fact that it guarantees you a loss. Nevertheless, the case above shows that you can violate both Finite Additivity and Normalization and yet not be licensed to accept any Dutch Books.

---

the example above, multiplying all your credences by -1 would yield a credence function involving negative credences but which still yields the same coherent fair betting quotients. However, as I show below, having negative credences guarantees exploitability. So just having coherent fair betting quotients is not sufficient to avoid exploitability.

[9] For instance, you are not licensed to accept a bet on $\{w_1, w_2, w_3\}$ which costs you money if $\{w_1, w_2, w_3\}$ is true, since you assign positive credence to that proposition and zero credence to its negation, and so in the expected value calculation, losses incurred if $\{w_1, w_2, w_3\}$ is true could not be compensated for by gains made if its negation, $\emptyset$, is true. Similarly for $\{w_1\}$ and its negation, $\{w_2, w_3\}$, and for $\{w_1, w_2\}$ and its negation, $\{w_3\}$.

6 An Objection: Credences Constitutively Linked to Fair Betting Quotients?

I have been employing expected utility theory to determine which actions, including actions related to betting, are licensed by certain credences. A credence function licenses you to take a bet, or perform any other sort of action, just in case that bet or action has highest expected utility. This is the case even for incoherent credence functions. Given this decision-theoretic perspective, fair betting quotients needn't match credences when those credences violate Negation Coherence, and some violations of Negation Coherence yield invulnerability to Dutch Books.

One might object to this argument and defend the DBA by rejecting this blanket application of expected utility theory. One might argue that credences are in fact constitutively linked to fair betting quotients, so that it's part of how we should use the term 'credence' that fair betting quotients always match credences.[10] On this sort of view, having credence n in A always licenses you to accept either side of a bet on A at $n : 1 - n$ odds, independently of any facts about expected utilities.

If you accept this view, then my objections to the DBA do not apply; different views about credences yield different results about betting behavior. But for my part, I prefer a conception of credences and their relation to action on which betting behavior is seen as just one type of behavior among many. I see no motivation for privileging betting behavior in particular. Both the decision-theoretic view I espouse and a view on which there is a constitutive connection between fair betting quotients and credences yield a tight link between credences and action. But a fully decision-theoretic view, on which credences license all types of behavior, whether betting-related or not, in the same way, is much more unified than a view on which credences license certain actions (accepting certain bets) in a manner independent of expected utility theory, and license other actions in a manner determined by expected utility theory. A view on which the relationship between credences and the actions they license is fully captured by a general decision theory is simpler

[10]This idea has its roots in de Finetti (1937), albeit with a more behavioristic understanding of fair betting quotients. For de Finetti, your credence in A is n just in case you are actually willing (as opposed to merely licensed) to accept either side of a bet on A at $n : 1 - n$ odds. The claim that there is a constitutive connection between fair betting quotients and credences is also found in contemporary discussions of the DBA, such as Maher (1993) and Howson and Urbach (1993).

and more attractive than one which credences can license different actions in very different ways.

7 Where do We Go From Here?

I have shown that a careful look at how credences license actions reveals the limitations of the DBA. The DBA cannot be used to condemn violations of Negation Coherence as irrational. Worse, incoherent credence functions can provably fail to license accepting any Dutch Books, provided that they violate Negation Coherence.[11] Hence, the DBA can at best show that, once you satisfy Negation Coherence, any deviation from probabilistic coherence elsewhere in your credence function is irrational.

Therefore, in order to have an argument that all rational agents have coherent credences, we must at least supplement the DBA with a separate argument that all rational agents satisfy Negation Coherence. How might this be done?

7.1 Impossible to Violate Negation Coherence?

One approach would be to argue that it is in fact *impossible* to violate Negation Coherence, that it is a prerequisite for being an agent that one satisfy Negation Coherence. Then, one might say that all rational agents have coherent credences, where this is partly due to their *agency*, which ensures that they satisfy Negation Coherence, and partly due to their *rationality*, which ensures that they go on to fully satisfy coherence. Agency itself gets us to Negation Coherence, and the DBA gets us from there to full coherence.

As an analogy, consider Non-Negativity. Arguably, there is not a distinct norm of rationality that one not have negative credences. Rather, it is simply impossible to have such credences. It is a conceptual truth that credences (unlike utilities, say) have upper and lower bounds, since one's degree of confidence cannot exceed absolute certainty, and one's degree of confidence cannot fall below absolute certainty of falsehood. As a partly conventional matter, we choose to represent the lower bound with 0.[12] Therefore, one

[11] To be clear, it is not the case that all incoherent credence functions which violate Negation Coherence are invulnerable to Dutch Books. Only some are.

[12] There are, however, good reasons for using 0 as the lower bound on credences. In principle, we could set the lower bound on credences below 0, but this would require

cannot have negative credences simply because one's credence cannot be lower than absolute certainty of falsehood.

Might it also be impossible to violate Negation Coherence? Offhand, it seems possible to violate Negation Coherence, and not just due to thoughtlessness or stupidity. One might, for instance, adhere to a non-classical logic.

In intuitionistic logic, an atomic proposition is true only if there is a proof of that proposition, and the negation of a proposition is true only if there is a *reductio ad absurdum* of that proposition. Thus, the law of excluded middle fails, since there might be neither a proof nor a *reductio* of some given proposition. An adherent of intuitionistic logic might therefore assign some credence $n < 1$ to $A \vee \neg A$ and credences in A and $\neg A$ that sum to n. This person therefore violates Negation Coherence. (Assuming that intuitionism is in fact false, this person also violates Normalization, since $A \vee \neg A$ is in fact a necessary truth.)

In paraconsistent logic, famously defended by Priest (1979), it is possible for a proposition and its negation to both be true. So, $A \wedge \neg A$ is not a contradiction. But the law of excluded middle still holds. An adherent of paraconsistent logic might therefore assign some some credence $n > 0.5$ to A and credence $m > 0.5$ to $\neg A$, while still assigning credence 1 to $A \vee \neg A$. This person therefore violates Negation Coherence as well. (This person also violates Finite Additivity, since $P(A \vee \neg A) \neq P(A) + P(\neg A)$.)[13]

If this is right, then it is possible to violate Negation Coherence. And so the fact that the DBA cannot show that Negation Coherence is a requirement of rationality really is an important limit on the role the DBA can play in epistemology. In particular, it cannot rule out the acceptance of a non-classical logic as irrational.

Of course, this does not completely settle the matter, since not everyone will agree with the intuitive take on the credences of intuitionists and paraconsistent logicians I gave above. One might argue, for instance, that the intuitionist does not really doubt the truth of $A \vee \neg A$, but instead just doubts the truth of the contingent, metalinguistic proposition that the sentence '$A \vee \neg A$' expresses a truth (and similarly, *mutatis mutandis*, for the paraconsistent logician). Such a philosopher might then go on to hold that it is in fact impossible to violate Negation Coherence, perhaps by arguing that

---

considerable complexity elsewhere in the theory.

[13]See Field (2009) for an interesting discussion for the relationship between logic and rational constraints on doxastic attitudes, with a focus on non-classical logics.

attribution to an agent of credences that violate Negation Coherence would never be an optimal way of making sense of that agent's behavior. However, it remains unclear what sort of argument could show that violations of Negation Coherence are impossible without also showing that violations of coherence, *simpliciter*, are impossible. Stalnaker (1984), for instance, defends a view of propositional attitudes where what it is for agent to have some set of propositional attitudes is for the attribution of those propositional attitudes to be part of an optimal explanation and rationalization of that agent's behavior.[14] Stalnaker argues that on this view, it is impossible to have inconsistent beliefs or to fail to believe a necessary truth, and the same arguments would support the claim that it is impossible to have probabilistically incoherent credences.[15] So, someone like Stalnaker might argue that it is impossible to violate Negation Coherence, but only on grounds that suggest that it is impossible to violate coherence in any way, thus making the DBA wholly superfluous.

My purpose is not to ajudicate between these two positions. The debate over the proper theory of propositional attitudes is long-running, and I have nothing new to add to it here. I merely want to emphasize that there is some intuitive pull toward saying that violations of Negation Coherence are possible, even if certain theories of propositional attitudes say otherwise. Moreover, it is difficult to see how one might argue that violations of Negation Coherence are impossible, while maintaining that it is possible to violate coherence in other ways. In this way, I doubt whether it is possible to argue that violations of Negation Coherence are impossible without leaving the DBA with no work left to do.

7.2 Irrational to Violate Negation Coherence?

Even if violations of Negation Coherence are *possible*, it might be that they are irrational, albeit for reasons wholly independent of the DBA. Now, there

[14]This Radical Interpretation theory of propositional attitudes is also defended by Davidson (1973) and Lewis (1974).

[15]Stalnaker treats purported ignorance of necessary truths as a sort of metalinguistic ignorance, or ignorance of which proposition is expressed by a given sentence. And he treats alleged cases of inconsistent beliefs as involving *fragmentation* - the agent is represented as having multiple belief states, each of which is consistent and which govern the agent's actions in different behavioral situations. The fact that different belief states are operative in different behavioral situations gives the misleading appearance of inconsistent beliefs.

are many arguments that have been advanced which entail that Negation Coherence is irrational, but they do so only by entailing that all forms of incoherence are irrational, and thus serve to replace, rather than supplement, the DBA.

So, for instance, the Representation Theorem Argument, defended by Maher (1993), among others, says that it is a requirement of rationality that your preferences have a certain structure, and that if your preferences have that structure, then you are best represented as an expected-utility-maximizer with probabilistically coherent credences. Hence, if you are rational, then you will have coherent credences.

Scoring Rule Arguments, discussed by Joyce (1998), hold that for every incoherent credence function, there is some coherent credence function which scores better on some measure of expected accuracy. Together with an argument that it is rational to have credences which maximize this sort of expected accuracy, Scoring Rule Arguments say that it is irrational to have incoherent credences, since each incoherent credence function is rationally inferior (in virtue of having lower expected accuracy) than some coherent credence function.

While I do not find these arguments convincing, I do not want to get into the details of these arguments here, for they have already generated an extensive literature.[16] The important thing to note is just that these arguments purport to show not that Negation Coherence in particular is a requirement of rationality, but that full probabilistic coherence is a requirement of rationality. In this way, if you are convinced by one of these arguments, then you will think that the DBA is just superfluous.

Might there be some argument that only shows that violations of Negation Coherence are irrational, thus leaving the DBA with an important role to play in getting us from Negation Coherence to full coherence? I doubt it. I grant that violations of Negation Coherence often seem more bizarre than some other sorts of incoherence, since Negation Coherence is a constraint that applies only to pairs of propositions. In this way, it seems that it should be particularly easy to satisfy Negation Coherence, in just the way that it is easier to avoid blatantly contradictory beliefs (like believing both P and $\neg P$) than to avoid more subtly contradictory beliefs. But this consideration does not seem like a good argument that all violations of Negation Coherence

[16] See especially Meacham and Weisberg (forthcoming) for criticism of Representation Theorem Arguments and Gibbard (2008) for criticism of Scoring Rule Arguments.

are irrational. For one thing, you might violate Negation Coherence for principled reasons rather than due to sloppy thinking, at least if adherence to certain non-classical logics like intuitionism and paraconsistent logic means that you really do violate Negation Coherence. For another, the mere fact that Negation Coherence is relatively easy to satisfy doesn't mean that it is irrational to violate it. There are lots of things that are easy to do but are not rationally required.[17]

Of course, it remains possible that it is just a brute, unexplained requirement of rationality that you satisfy Negation Coherence. This would be somewhat unsatisfying, but unsatisfactoriness doesn't entail falsehood. Chains of explanations must come to an end somewhere, and perhaps the explanation of why it is irrational to have incoherent credences ends in the brute fact that it is irrational to violate Negation Coherence. I think that this is the position that must be adopted in order to give the DBA a role to play in showing that all rational agents satisfy probabilistic coherence. If we don't rule out violations of Negation Coherence as impossible or irrational, it is possible to have incoherent credences that cannot be shown by the DBA

[17]Actually, it may be possible to argue that violations of Finite Additivity in particular are irrational. And, since violating Negation Coherence entails violating either Finite Additivity, Normalization, or both, this claim would mean that some violations of Negation Coherence - those that stem from violations of Finite Additivity - are irrational. The idea is that violations of Finite Additivity yield inconsistent calculations of expected utility, and hence inconsistent behavioral commitments. Suppose A, B, and C are mutually exclusive and jointly exhaustive. You assign credence 0.2 to A, 0.2 to B, and 0.2 to C, but you assign credence 0.8 to $B \vee C$, thus violating Finite Additivity. Consider an act φ that yields utility 3 if A and utility -1 if either B or C is true. One way of calculating expected utility takes each of A, B, and C to be an outcome, so that the expected utility of φ is $0.2 \times 3 + 0.2 \times -1 + 0.2 \times -1 = 0.2$. And so φ looks like a good thing to do. But we could also take $B \vee C$ to be an outcome, since φ yields the same utility whichever disjunct is true, and therefore use your credence in the disjunction $B \vee C$ in the calculation. And so the expected utility of φ is $0.2 \times 3 + 0.8 \times -1 = -0.2$. On this way of doing things, φ looks like a bad thing to do. Since it should be equally legitimate to think of outcomes in a maximally fine-grained way (as on the first way of calculating expected utilities) and to think of them in a more coarse-grained way, lumping together propositions which are assigned the same utility (as on the second way of doing things), you could interpret this as a situation in which your credences commit you both to φ-ing (since φ has positive expected utility on one way of calculating expected utilities) and to not φ-ing (since it has negative expected utility on another way of calculating expected utilities). Hence, violations of Finite Additivity yield inconsistent behavioral commitments, and are therefore irrational. Still, this argument would not show that violations of Negation Coherence that stem from violations of Normalization are irrational.

to be irrational. But if we rule out violations of Negation Coherence as impossible or irrational only by ruling out *all* violations of coherence as impossible or irrational, then the DBA is superfluous. To maintain, then, that the DBA has an important role to play in showing that all rational agents satisfy coherence, we are forced to hold that it is just a brute requirement of rationality that you satisfy Negation Coherence.[18]

8 Conclusion

I have argued that if we think of credences as licensing actions only in virtue of expected utility considerations, then the DBA cannot be used to condemn violations of Negation Coherence as irrational. Worse, it is actually possible for incoherent credences to fail to license accepting any Dutch Books, provided those credences violate Negation Coherence. Hence, the DBA can at best show that, once one satisfies Negation Coherence, any deviation from probabilistic coherence is irrational. In this way, the DBA is a partial result and falls short of showing that all incoherent credences are irrational.

Then, unless we are prepared to accept that it can in some cases be rationally permissible to have incoherent credences, we face a dilemma. First horn: It is impossible to be incoherent in any way. Reasons for thinking that it is impossible to violate Negation Coherence generalize to suggest that it is impossible to violate coherence at all. Second horn: Some structural constraints on credences (in particular Negation Coherence) are brute requirements of rationality that must be accepted for their intuitive, pretheoretic plausibility rather than on the basis of any argument. No independent arguments for the irrationality of violating Negation Coherence are compelling.[19]

[18]Perhaps accepting Negation Coherence, and possibly even all of probabilistic coherence, as a brute requirement of rationality would not be so bad after all. Compare the requirement that binary beliefs be logically consistent. We have no Dutch Book-style exploitability argument for this requirement of rationality. You might think that beliefs must be logically consistent, because if your beliefs are inconsistent, you are guaranteed to have at least one false belief, and that is bad. But this is not a compelling argument, since if your beliefs are inconsistent (and closed under entailment), you are also guaranteed to have at least one true belief, and that is good. So, I doubt whether there is any compelling argument for the requirement that beliefs be consistent. Instead, it is a pretheoretically compelling requirement of rationality. I propose that we should think of Negation Coherence in the same way.

[19]This assumes, of course, the failure of Representation Theorem and Scoring Rule

Therefore, provided that it is so much as *possible* to have incoherent credences, some structural constraints on credences must be brute rational requirements.[20]

References

Christensen, David. "Dutch-Book Arguments Depragmatized: Epistemic Consistency for Partial Believers." *Journal of Philosophy* 93 (1996): 450–479.

Davidson, Donald. "Radical Interpretation." *Dialectica* 27 (1973): 314–328.

Finetti, Brunode . "La Prevision: ses lois logiques, ses sources subjectives." *Annales de l'Institut Henri Poincar* (1937).

Field, Hartry. "What is the Normative Role of Logic?." *Aristotelian Society Supplementary* 83 (2009): 251–268.

Gibbard, Allan. "Rational Credence and the Value of Truth." *Oxford Studies in Epistemology, Vol 2.* . Oxford University Press, 2008.

Hajek, Alan. "Dutch Book Arguments." *The Oxford Handbook of Rational and Social Choice.* . Oxford University Press, 2008.

Howson, Colin and Peter Urbach. *Scientific Reasoning: The Bayesian Approach.* Open Court Publishing Company, 1993.

Joyce, James. "A Nonpragmatic Vindication of Probabilism." *Philosophy of Science* 65 (1998): 575–603.

Kemeny, John. "Fair Bets and Inductive Probabilities." *Journal of Symbolic Logic* 20 (1955): 263–273.

Kolmogorov, Andrey. *Grundbegriffe der Wahrscheinlichkeitsrechnung.* Julius Springer, 1933.

Arguments. Again, I reject these arguments for reasons given in Meacham and Weisberg (forthcoming) and Gibbard (2008), which cannot be recounted here for reasons of space.

[20]I thank Ryan Doody, Dan Greco, Caspar Hare, Vann McGee, Agustín Rayo, Robert Stalnaker, Roger White, an MIT Work in Progress audience, and an anonymous referee for their helpful feedback and discussion.

Lehman, R. Sherman. "On Confirmation and Rational Betting." *Journal of Symbolic Logic* 20 (1955): 251–262.

Lewis, David. "Radical Interpretation." *Synthese* 27 (1974): 331–344.

Lewis, David. *Papers in Metaphysics and Epistemology.* Cambridge University Press, 1999.

Maher, Patrick. *Betting on Theories.* Cambridge University Press, 1993.

Meacham, Christopher and Jonathan Weisberg. "Representation Theorems and the Foundations of Decision Theory." *Australasian Journal of Philosophy* (forthcoming).

Priest, Graham. "The Logic of Paradox." *Journal of Philosophical Logic* 8 (1979): 219–241.

Skyrms, Brian. "Dynamic Coherence and Probability Kinematics." *Philosophy of Science* 54 (1987): 1–20.

Stalnaker, Robert. *Inquiry.* The MIT Press, 1984.

Teller, Paul. "Conditionalization and Observation." *Synthese* 26 (1973): 218–258.

Fraassen, Basvan . "Belief and the Will." *Journal of Philosophy* 81 (1984): 235–256.